

# Modelling Directional Variance and Variograms Using Geo-Optical Models

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**Abstract** The image BRDF of forests and woodlands is a statistical function which operates at the scale of an average patch of cover rather than at the scale of crowns. Studies of the image Hotspot effect using aerial photography and high spatial resolution scanner data shows very high variance at this detailed scale. In addition, the directional effects of the sun and observer positions interact significantly to create an angular anisotropic variation which persists up to aggregated scales. This has been called the BRVF or Bidirectional Reflectance Variance Function. There has been a recent growing interest in the directional variance or variogram structure of high resolution images as a means to interpret structure and such data have become regularly flown. This work is an extension of the use of image variance to interpret structure as pioneered by Strahler and Li and explored by various authors over the last 15 years. Directional variograms and BRVF functions for forests at the crown scale can be computed using approximations to the overlap functions driving these second order statistics and compared with numerically integrated simulations. It has been shown that the scaling of BRVF and the anisotropy introduced into the variogram can be accurately modelled.

**Key words** Geometrical optical models, Canopy BRDF, Image variance, Variogram

## 1 INTRODUCTION

High local spatial variance and large, anisotropic change in recorded radiance with changing sun position and sensor view angle are characteristic of images of woodlands and open forest areas. In these ecosystems, the canopy is discontinuous, or 'gappy', and the local spatial and view angle variation in images is created by a number of ecologically significant factors. These include interactions between the discrete nature of the canopy, the high natural spatial variation in canopy structure and the visible shadowing effects as the sun and view positions vary. The work reported here addresses how this variation can be modelled to estimate structure in some simple canopies. Structural effects are often dominant in the evolution of vegetation cover and the structural parameters (for example, tree and shrub crown sizes and heights, leaf area, density and distribution as well as grass cover patchiness) must be known if the evolution of the system is to be effectively monitored

or predicted.

The work described uses a set of models (called Geometrical Optical, or GO models) for the changes observed in scene reflectance as the view and sun direction change. In the form used here, these were outlined by Jupp *et al.*<sup>[1]</sup>, Strahler and Jupp<sup>[2]</sup> and Li and Strahler<sup>[3]</sup>. A summary of the implications they have for the study of canopy structure is presented by Jupp and Walker<sup>[4]</sup>. These resulting scene 'brightness' variations so modelled are expressed in the Bidirectional Reflectance Distribution Function (BRDF). The opportunity these models provide for structural measurement is that the BRDF and the scene hotspot function (which is one component of the BRDF) contain significant information relating to canopy structure and scales up in a way that makes it equally useful for measurements from aircraft<sup>[5]</sup> and satellite<sup>[6]</sup> platforms.

Geometrically based BRDF and hotspot effects for soils were established by Hapke<sup>[7,8]</sup>. For vegetation, various approximate and empirical hotspot functions have been developed by Kuusk<sup>[9]</sup>, Gerstl *et*

al.<sup>[10]</sup>, and Nilson and Kuusk<sup>[11]</sup>. Explicit geometrically based models for the hotspot effect for tree crowns were presented by Nilson<sup>[12]</sup>, Jupp *et al.*<sup>[1]</sup> and Strahler and Jupp<sup>[13]</sup>. The use of explicit geometric optics for leaf canopies was outlined by Jupp and Strahler<sup>[2]</sup> and extended to include leaf shape and stem effects by Qin<sup>[14]</sup> and others as well summarised and evaluated by Qin and Goel<sup>[15]</sup> and Qin *et al.*<sup>[16]</sup>

The benefit of the geometric approach is illustrated in the development of relationships between the hotspot size and shape and scale-free ratios such as crown width to height and diameter to depth in the case of trees and leaf diameter to height and leaf length to width in the case of canopies. These structural ratios define the hotspot component of the BRDF even when the pixel size is very large relative to the sizes of crowns and leaves. Geometrical models have also been used to investigate the way image variance and covariance change as scale changes and how much information the covariance data from the image provides about the vegetation structure in the underlying scene being imaged. This work uses the tools relating scene and image models and scale described by Jupp *et al.*<sup>[17,18]</sup>, Jupp and Woodcock. The change in variance with view has been observed empirically, but the work described here provides an ability to model this 'BRVF' (Bidirectional Reflectance Variance Function) and assess its value for determining the land surface structure.

The changing mean and variance effects which arise when the same scene is viewed from different directions and with different sun positions lead to a significant image 'brightness' and texture variation issue in remote sensing. The images concerned arise from aerial photography, airborne scanners and video systems and will derive from the newer satellite borne sensors such as those of EOS. The changing means and variance/texture must either be incorporated into new methods of image analysis or (at least) taken into account before traditional image processing is applied to (mosaics of) images with widely varying look and sun angle combinations.

## 2 GEOMETRICAL OPTICAL SCENE MODELS FOR FORESTS AND WOODLANDS

In the Geometrical Optical (GO) model for forests and woodlands<sup>[19,20]</sup>, there are four kinds of ground cover 'visible' from a given direction. These are referred to as scene components and consist of sunlit canopy (symbol  $C$ ), shaded canopy ( $T$ ), sunlit background ( $G$ ), and shaded background ( $Z$ ). Each component is assumed to have a characteristic radiance and the radiance of a pixel is modelled as the area weighted combination (or linear mixture) of the characteristic component radiances. That is, the observed radiance of a single pixel ( $r_s$ ) is modelled as:

$$r_s = k_C R_C + k_T R_T + k_G R_G + k_Z R_Z$$

where  $C$ ,  $T$ ,  $G$ , and  $Z$  indicate the radiances of the four components as named above,  $R_x$  represents the (mean) radiance of component 'x' and  $k$  indicates the sensed proportion of each component within the pixel from the given view direction. Since there are only the four components:

$$k_C + k_T + k_G + k_Z = 1$$

The mean radiance over the scene ( $R_s$ ), assuming the view and sun directions are constant, can be written as:

$$R_s = K_C R_C + K_T R_T + K_G R_G + K_Z R_Z$$

where, capital  $K_x$  represents the mean or expected value of the varying proportions  $k_x$  over the scene. This mean value, as a function of sun and observer position, defines the BRDF of the scene.

In order for the scene BRDF model to be computed, a description of the size and shapes of the objects, their density and how they are distributed over the background is needed and the geometrical relationships between the objects and the four components must be established. Jupp *et al.*<sup>[1]</sup>, Jupp and Strahler<sup>[2]</sup> and Li and Strahler<sup>[3]</sup> describe such a model for spheroids which is valid for any view (subscript  $v$ ) or illumination (subscript  $i$ ) angles using the 'Boolean' model of Serra<sup>[21]</sup>. In the Boolean model, the object 'centres' are assumed to be randomly distributed in a 'Poisson' distribution with density  $\lambda$ .

By defining the geometry and the distributions, expressions for the  $K_x$  may be derived. In general<sup>[2]</sup>, the proportions have the following relationships:

$$K_C + K_T = 1 - e^{-\lambda A_v}$$

and

$$\begin{aligned} K_G &= e^{-\lambda(A_i + A_v - O_{iv}(\theta_i, \theta_v, \phi_p))} \\ &= e^{-\lambda \bar{A}_\Gamma} \end{aligned}$$

where  $A_i$  is the projected shadow area of a crown (not necessarily spheroidal) from the sun (ie illumination) direction,  $A_v$  is the projected 'shadow' (or obscured) area from the sensor (ie, view) direction and  $O_{iv}$  is the area of overlap between the projected 'shadows' of the object on the background from the two directions, which depends on the sun and view zenith angles ( $\theta_i, \theta_v$ ) and the azimuthal difference between them ( $\phi_p$ ).

The projected 'shadow' overlap function  $O_{iv}(\theta_i, \theta_v, \phi)$  depends on the (unitless) ratios  $D/h$  and  $b/D$  where  $h$  is the height of the trees and  $D$  is the crown diameter<sup>[1,13]</sup> as well as the illumination and view directions. The quantity  $\bar{A}_\Gamma$  is the composite area of the object formed by the union of the sun and observer 'shadow' areas (see Fig. 1) and the scene behaves like a Boolean model with this composite primary 'grain'.

to pixel behaviour of the image is quite well described by a simpler form of the model in which the shaded background, sunlit (but still relatively dark) tree and shaded tree components are combined into one so that;

$$r_s = R_X + k_G(R_G - R_X)$$

where  $X$  is a composite component combining sunlit and shaded tree and shaded background. This simpler model has the advantage for this discussion that it shows how, in many woodland areas, the image pixel to pixel variation is driven primarily by the variation in the proportion of sunlit background which is visible in the pixels and the contrast between this sunlit background and the other components. It also provides a simple estimate for  $k_G$  from images where  $R_G$  and  $R_X$  are known for an appropriate image channel as:

$$k_G = \frac{R_s - R_X}{R_G - R_X}$$

For such a model, the mean radiance (ie BRDF) over all pixels in a patch with the same basic underlying type of cover and structure is therefore;

$$E(r_s) = R_s = R_X + (R_G - R_X) K_G$$

where  $K_G$  is the mean value of  $k_G$ , or the expected proportion of visible sunlit background. Moreover, the variance of the pixel radiance is;

$$\text{var}(r_s) = (R_G - R_X)^2 \text{var}(k_G)$$

from which it is clear that variance of a scene, for which this simplified form of the model is appropriate, is defined by that of  $k_G$  and the contrast between the sunlit background and the composite of tree and shadow.

In the simplified model, the viewed scene consists of objects made up of a composite of projected tree plus shadow silhouettes scattered over a sunlit background. The sensor integrates the radiance over a pixel so the model for the resulting image is one of objects on a contrasting background regularised by the integration into pixels. Such models may be handled by the tools described in Jupp *et al.*<sup>[17,18]</sup> from which expressions for the way  $\text{var}(k_G)$  changes with pixel size can be derived. In general, the expressions are quite complex and must be computed numerically. General expressions for the variance have also been

Fig. 1 Overlap function driving BRDF of forest canopy

In the woodlands and open forest areas typical of the area of Australia where the model studies are being made and in appropriate spectral bands, the pixel

used to invert structure through the Li Strahler model<sup>[19, 20, 22]</sup>.

### 3 INVESTIGATING CANOPY VARIANCE & COVARIANCE

The pixel data change significantly with view position and the variance (also the spatial covariance) changes with both view angle and pixel size. Even an (apparently) uniform area of forest cover will therefore create significant variation in both brightness and texture in airborne scanner data. The variation in the mean is effectively what has been termed the 'BRDF' (or Bidirectional Reflectance Distribution Function) and our work suggests that attention should also be given to the 'BRVF' or Bidirectional Reflectance Variance Function for situations where multiangle viewing is possible. The variance and variogram are

also dependent on pixel size and, since their expressions are rather complicated, must generally be computed numerically.

In experiments discussed by Jupp and Woodcock<sup>[23]</sup>, low level aerial photography was used to investigate the BRDF of a woodland area in Australia at high spatial resolution. It was clear from this experiment that the spatial variance was extremely high. By compositing frames of photography it was possible to extract a stable mean BRDF and show it could be modelled adequately by the simple model described above. However, the number of frame that would be needed for the variance to become low was found to be quite large. In other more recent work, Pickup and *et al.*<sup>[24]</sup> have had to stack up to 30 video frames for the BRDF related frame brightness variation to stabilise. In that case, the objective was to 'normalise' the run of video frames and remove the frame brightness.

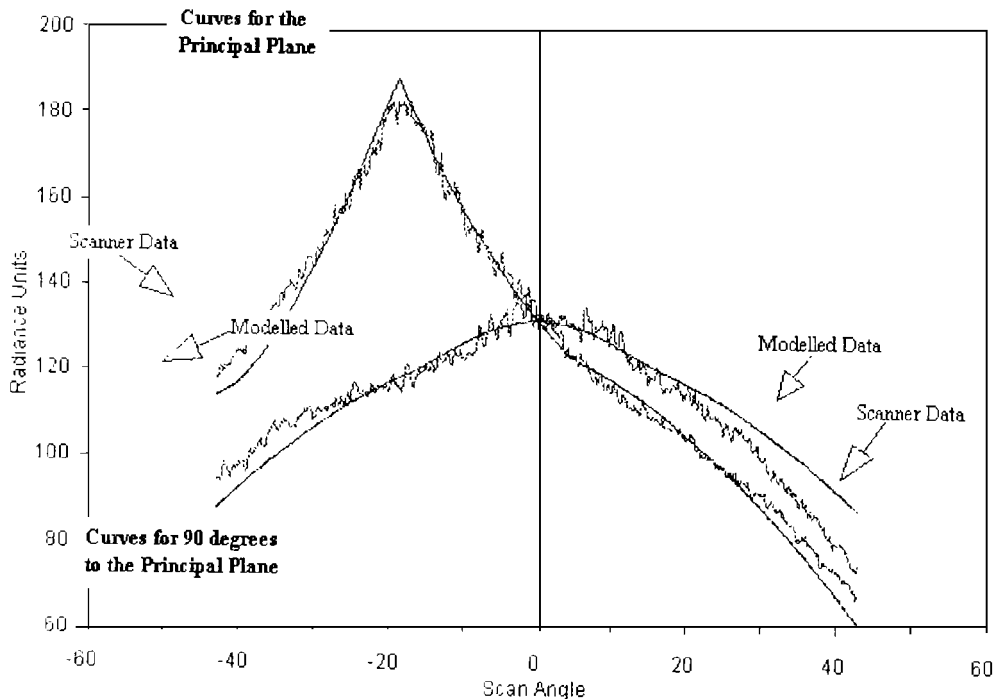


Fig. 2 Fig. 1 BRDF (for SWIR) along the Principal Plane and 90 degrees to Principal Plane modelled data compared to DATM scanner data

As an alternative method of investigating BRDF where the averaging could be made very high, a Daedalus scanner was flown over the same forest in

The data were flown at three altitudes to obtain nominal pixel sizes of 2.5 metres, 5 metres and 10 metres. In the case of the 2.5 and 5 metres cases,

runs were made at right angles so that the scanner was scanning into and at 90° to the Principal Plane. By averaging long runs of lines where the scanner flew over relatively homogeneous land covers, the mean BRDF and the residual BRVF could be obtained. Fig. 2 shows the fit to the BRDF obtained by averaging more than 1000 lines for the 5 metre data. Fig. 3 shows the standard deviations for the flights. Clearly, as the pixel size increases (aircraft altitude increases) the variance decreases.

ple form :

$$\sigma^2 = K_G(1 - K_G)$$

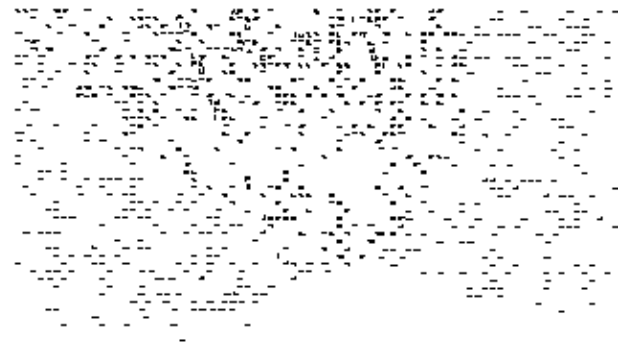


Fig. 4 Overlap function driving scale variance of a forest

When the data are averaged into pixels, the work outlined in Jupp *et al.*<sup>[17,18]</sup> which is based on the ideas of regularisation described in Serra<sup>[21]</sup> allows us to write:

$$\begin{aligned} \gamma_Z(h) &= \sigma_Z^2 - H_Z * Cov_f(h) \\ \sigma_Z^2 &= H_Z * Cov_f(0) \end{aligned}$$

where  $Z$  represents the pixel area,  $H_Z$  is the pixel overlap function and

$$H_Z * Cov_f(h) = \int_R H_Z(h+p) Cov(p) dp$$

In order to approximate the variation observed in Fig. 3, it might seem to be possible to approximate the composite area by a disk with the same area and the same mass centre. If this is done, and the view and observer directions are (at least locally) constant, the variance may be approximated by the simple expressions for the disk model given by Jupp *et al.*<sup>[18]</sup>. If this is done, the expected variances at the different positions in the image and different view and sun angles are shown in Fig. 5. It is clear that the variation and scaling found in Fig. 3 is being well explained. It is also interesting how for the cover being modelled in this area (40%) that the variance is high in the vicinity of the hotspot at the finest pixel scale but becomes much lower than its neighbourhood when the pixel size reaches 10 metres. This effect can be seen in the images as well as in comparisons of the low and high altitude aerial photography taken previously.

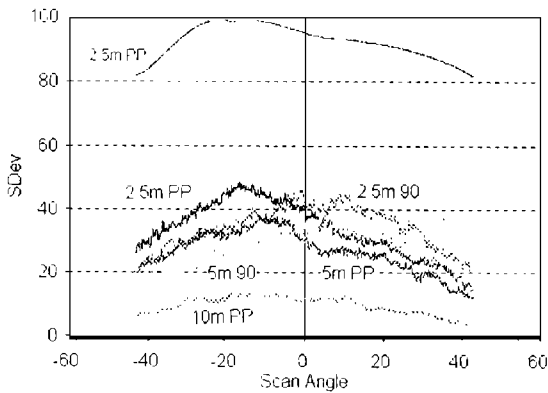


Fig. 3 Angular variation of standard deviation for daedalus data from goonoo

To model the variance and its scaling properties, we can use the work reported by Jupp *et al.*<sup>[17,18]</sup> with the geometric Boolean model derived above in the simple case. That is, the probability of hitting a sunlit patch in the simple model is:

$$K_g = e^{-\lambda \bar{O}^*}$$

and the point covariance and variogram can be written as:

$$\begin{aligned} cov(h) &= K_G^2(e^{\lambda \bar{O}^*(h)} - 1) \\ \gamma(h) &= K_G^2(1/K_G - e^{\lambda \bar{O}^*(h)}) \end{aligned}$$

where  $h$  is a vector and  $\bar{O}^*(h)$  is the area of overlap between the composite union of projected tree 'shadow' from the view and sun directions and itself shifted by Serra<sup>[21]</sup>, Jupp *et al.*<sup>[17,18]</sup>, which is illustrated in Fig. 4. When there is no averaging into pixels, the variance (the point variance) has the sim-

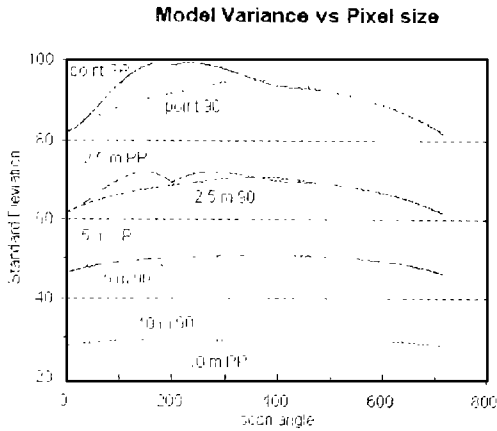


Fig. 5 Modelled scaled angular variance of daedalus data for nominal pixel sizes

However, this approximation is obviously deficient if for no other reason that it does not describe the anisotropy clearly visible in images of trees with their strongly directionally cast shadows. To model the full directional and anisotropic effect it is necessary to compute the complex overlap function. This may be done but some of the final expression must be computed numerically. The result for the point covariance of trees with cover and structure chosen to be similar to that described by Woodcock *et al.*<sup>[23]</sup> is shown in Fig. 6. Comparing Fig. 6 with the directional variograms by Woodcock *et al.*<sup>[23]</sup> shows how

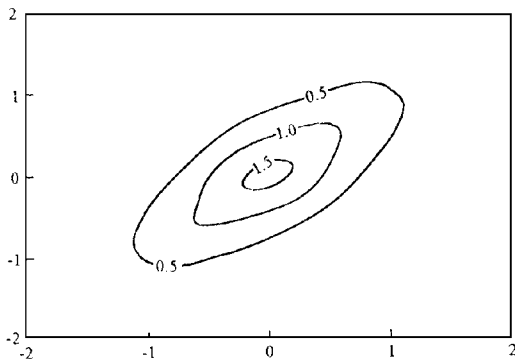


Fig. 6 Modelled anisotropic multi-directional variogram for Forest data

the anisotropy can be well captured by the model. There has been some recent interest in empirical directional variograms in forest studies using low level scanner data outlined by St-Onge and Cavayas<sup>[25]</sup>. In that study the directional variogram was modelled empirically. This paper has illustrated the underlying geometric model on which the anisotropy is based. It is clearly a strong indicator of structure. How it scales will be discussed in future work.

## 4 CONCLUSIONS

Using angular variations (the BRDF and BRVF) derived from multi-sun and observer angle sensing of the earth offers the possibility of monitoring structural changes at a finer grain than image resolution within broad mosaics of different covers. Data of this type are currently available from aerial photography and airborne scanners (such as the DATM, CASI, airborne Video systems and ASAS). The results obtained to date are also useful for interpreting AVHRR data. In the future, MISR and POLDER on the EOS platform will enable multiangular space data to be analysed. In particular, the hotspot model, with its scaling properties, offers a new opportunity to monitor land surface structure from space. Unlike much currently operational image processing, however, its complete exploitation depends on achieving a balance between empirical study and physical modelling.

Models for the resulting primary statistics of images—the Bidirectional Reflectance Distribution Function (BRDF), the Bidirectional Reflectance Variance Function (BRVF) and the anisotropic directional variogram have been tested using low altitude, wide angle photography and airborne scanner data from an Australian open forest area. The conclusion is that the models can explain the angular variation at a variety of scales. This variation and the (anisotropic) variation in radiance in remotely sensed images and its spatial variance and scaling effects, which are created by the combination of land surface structure, the sun position and view angle, provide a significant opportunity for detecting temporal changes in woodland and open forest cover and structure. The BRDF models

have been considerably simplified recently through the 'kernel' models outlined by Wanner and Strahler<sup>[26]</sup> and are providing practical tools for balancing mosaics of airborne scanner and video data. However, the different textures which accompany the changes in scene 'brightness' also need parallel work and simplification.

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## 用几何光学模型对方向变化和直方图建模

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**摘要** 森林和林地的图象二向性反射函数是一个统计函数，较之小尺度的树冠，它更多地用于大尺度的均匀覆盖的地块。用航空象片和高空间分辨率扫描仪数据作的图象热点影响研究在小尺度下显示出很大方差，而且，太阳和观测角度的交互变化进一步增加了这种反射各向异性变化的方差并有规律地继续呈现在分辨率低的图象中，这被称作 BRVF 或二向性反射方差函数。近年来，作为一种解释结构的手段，高分辨率图象的方向性方差和直方图结构越来越受到重视，这方面的数据也越来越多。这项工作是利用图象方差来解释结构问题 (Strahler 和李小文倡导) 的一种延伸，并在过去 15 年中由众多人员作了大量工作。在树冠尺度下，森林的直方图和二向反射方差函数可以计算出来，这里利用了近似迭代函数来处理这些数据并和数值积分模拟进行了对比，结果显示可对二向性反射方差函数的测量和引入直方图的各向异性进行准确建模。

**关键词** 几何光学模型，冠层二向性反射分布函数，图象方差，直方图